647510 **BSCmaMJ205x-3C** **Seat No :________**

B.Sc. Semester - 3 (NEP-2020) Examination
OCT/NOV-2025 (AS PER SYLLABUS YEAR JUNE 2025)
MATH: Linear Algebra -1 (Major-5)

Time: 2:00 Hours **Marks:50**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Que.1 Attempt any two out of three [10]

1) Prove that the set $V = \{(a,b): a \in \mathbb{R}, b > 0\}$ with following maps is a vector space, $(a,b) + (c,d) = (a+c, bd)$ and $\alpha(a,b) = (\alpha a, b^{\alpha})$; $\forall (a,b),(c,d) \in V \& \ \alpha \in \mathbb{R}$.
2) (a) Prove that the set $\{sinx, cosx, \cos(x - \frac{\pi}{6})\}$ is linearly dependent in $C[0,2\pi]$.
   (b) Check whether the subset $W = \{(a,b,c): 2a + 5b - c = 0\}$ of $\mathbb{R}^3$ is a subspace or not.
3) Prove that super set of a linearly dependent set is always linearly dependent.

Que.2 Attempt any two out of three [10]

1) Find the condition for set $A = \{(x,-1,-1), (-1,x,-1), (-1,-1,x)\}$ to be a base of $\mathbb{R}^3$.
2) Prove that there is a basis for each finite dimensional vector space.
3) Verify dimension theorem for subspaces $W_1 = span\{(1,1,0),(0,1,1)\}$ and $W_2 = span\{(1,1,0),(0,-1,1)\}$ of $\mathbb{R}^3$.

Que.3 Attempt any two out of three [10]

1) Find a linear map if exist, $T: \mathbb{R}^2 \to \mathbb{R}^2$ such that $T(1,2) = (3,0)$ and $T(2,1) = (1,2)$.
2) State and prove rank nullity theorem.
3) In usual notation, prove that a linear map $T: U \to V$ is one-one iff $N_T = \{\theta\}$.

Que.4 Attempt any two out of three [10]

1) Define an inner product space and check whether the following is an inner product on $\mathbb{R}^2$ or not: $< (x_1,x_2) \cdot (y_1,y_2) >= {x_1}^2 - 2x_1y_2 - 2x_2y_1 + {y_1}^2$
2) State and prove Cauchy-Schwartz inequality.
3) If $\|u+v\| = \|u\| + \|v\|$ then prove that $u \ and \ v$ are linearly dependent, where vectors $u \ and \ v$ are in inner product space.

Que.5 Attempt any two out of three [10]

1) Prove that the intersection of two subspace of a vector space is also a subspace. Is union of two subspace of a vector space subspace ?
2) Let $T: \mathbb{R}^3 \to \mathbb{R}^2$ be a linear map defined as $T(a,b,c) = (2a+b+c, 3a+2b+c)$, $\forall (a,b,c) \in \mathbb{R}^3$ and let $B_1 = \{(1,2,3),(4,5,0),(6,0,0)\}$ and $B_2 = \{(2,7),(5,0)\}$ be a basis of $\mathbb{R}^3$ and $\mathbb{R}^2$ respectively. Then find $[T; B_1, B_2]$.
3) Let $\{(1,2,3),(4,5,0),(6,0,0)\}$ be a basis of an standard inner product space $\mathbb{R}^3$. Find an orthonormal basis of $\mathbb{R}^3$ from given basis by Gram Schmidt process.

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